Closing Tues: HW 9.7(2), 9.8, 9.9

Exam 1 is Thur, Jan. 25th covers 9.3 - 9.9.

Recall: Finding Derivatives

Step 0: Rewrite powers and simplify.

Step 1: Product, Quotient or Chain?

Step 2: Use appropriate rule, in the middle of that rule you may need to do a derivative (back to step 1)

Entry Task: Find the derivatives of

1. $f(x) = 4x^3(6x^2 + 7)^{10}$
2. $g(x) = \dfrac{3}{\sqrt{x}} + \dfrac{x^2}{5} + \dfrac{x^4}{3x + 1}$

9.8 The Second Derivative

The *second derivative* is the derivative of the derivative. We denote it

$$f''(x) \quad \text{or} \quad \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

Example:

Assume *x* in seconds and *y* in feet.

$$y = x^3 + 2x$$

The second derivative represents the rate at which the *rate* of the original quantity is changing.

(*rate of change of rate of change*)

We will interpret more later, for now compute it.

Example: Find $f''(x)$
(use only positive exponents in your final answer)

$$f(x) = \frac{2}{x^3} - \sqrt{x}$$

Example: Revenue (in dollars) is given by

$$R(x) = 70x + 0.4x^3$$

if you sell x items.

1. What is marginal revenue (denoted *MR* or $\overline{MR}$) at 10 items?
2. What is the rate of change of marginal revenue when you sell 10 items?

9.9 Applications and Deriv. Graphs

***Application 1*: Demand Functions and Total Revenue**

Recall from Math 111:

A *demand curve is* given by an equation that relates the *quantity* that will sell based on the market *selling price*.

And

$$\text{Revenue} = \text{Price} \cdot \text{Quantity}$$

Example (from HW 9.9/1):

Assume *x* = items sold (quantity)

p = price per item.

From market analysis you estimate

$$p = 460 - 0.2x$$

a. Find TR(x) and MR(x).

b. What quantity maximizes TR(x)?

Two More Examples:

1. (part of HW 9.9/6)
   The selling **price** on the competitive market is 90 dollars/item.
   Find $TR(x)$ and $MR(x)$.

2. If the demand function is

$$p = \frac{500}{(3x+1)^2}$$

   Find $TR(x)$ and $MR(x)$.

***Application 2*: Cost Analysis and Profit**

Recall from Math 111:

TC(x) = “total cost to produce x items”

AC(x) = “overall average cost to produce *x* items”

$$AC(x) = \frac{TC(x)}{x} \text{ and } TC(x) = x\,AC(x)$$

Example (part of HW 9.9/5 and 6):

The (average) **cost** per unit is given by

$$130 + 0.5x \quad \text{dollars/item}$$

Find $TC(x)$ and $MC(x)$.

Recall from Math 111:

Profit and marginal profit are given by

$$P(x) = TR(x) - TC(x)$$
$$MP(x) = MR(x) - MC(x)$$

When profit is maximized

$$MR(x) = MC(x)$$

Specifically, where it switches from MR > MC to MR < MC.

Example (*directly* from HW 9.9/5)

The price of a certain product is $400. The cost per unit of producing the product is 130 + 0.5x dollars/item.

a. Find $TR(x)$ and $MR(x)$.
b. Find $TC(x)$ and $MC(x)$.
c. Find $P(x)$ and $MP(x)$.
d. How many units should you produce and sell to maximize its profits?

Another example
(*directly* from an old midterm):
You sell items.
If q is in **hundred items**, then TR(q) and TC(q) in **hundred dollars** are given by

$$TR(q) = 30q$$
$$TC(q) = q^3 - 15q^2 + 78q + 10$$

a. Find marginal cost at 2 hundred items

b. Find the longest interval over which marginal revenue exceeds marginal cost.

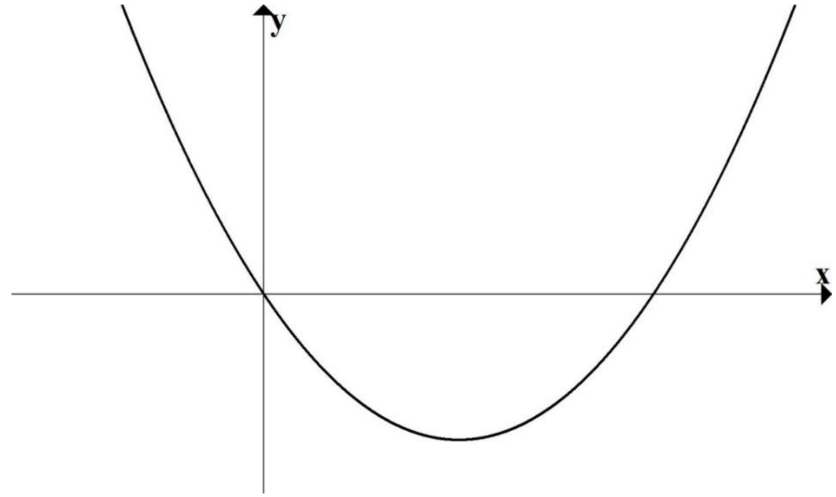
c. What is the maximum value of profit?

Graphs and Derivatives

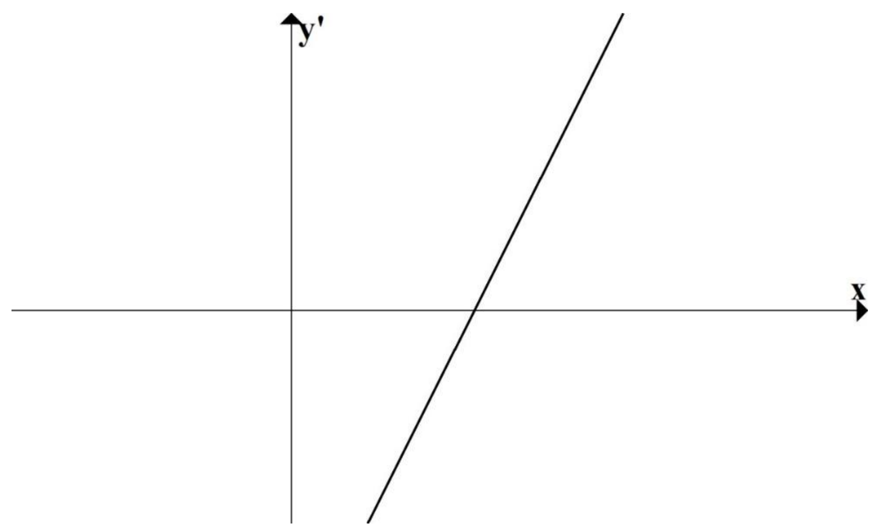
Example: Let $f(x) = 2x^2 - 3x$

Find $f'(x)$.

$f(x) = 2x^2 - 3x$



$f'(x) = 4x - 3$



Notes/Observations: Given $y = f(x)$.

- $y = f'(x)$ is a new function.
- $f(x) =$ "height of the graph at *x*"
- $f'(x) =$ "slope of $f(x)$ at x"
- $f'(x)$ is "instantaneous rate of change" (speedometer speed)
- The units of $f'(x)$ are $\frac{\text{y-units}}{\text{x-units}}$.

Fundamental to all applications:

$f(x)$	$f'(x)$
Horiz. Tangent *(peak, valley, or "chair")*	**Zero** *(crosses x-axis)*
Increasing (*uphill*)	**Positive** (*above x-axis*)
Decreasing (*downhill*)	**Negative** (*below x-axis*)

Old Exam Question:

The height of a balloon after t seconds is given by

$$B(t) = 15t^2 - t^3 \qquad \text{feet.}$$

a. At time t = 1 second, is the balloon rising or falling?
b. Find the maximum height reached by the balloon.